DESIGN OF FRACTIONAL ORDER SLIDING MODE CONTROLLER FOR 2 DOF HELICOPTER SYSTEM

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Received: 05th April 2018 Accepted: 07th July 2018

ABSTRACT

This paper presents a real time control application for a prototype helicopter system by using Sliding Mode Control strategy modified with small differential deviations. Experimental and simulation studies demonstrate the performance of the proposed controller for a class of flight systems with unstable dynamics. It is shown that the proposed controller decreases the steady state errors of the vertical and horizontal motion of the system. Small differential deviations strengthen or weaken the effect of the differential to obtain better tracking response in case of external disturbances. Numerical and experimental results match each other demonstrating the tracking of desired rotor angles and suppression the disturbance effects.

Keywords: Fractional Calculus, Sliding Mode Control, 2 DOF Helicopter System, TRMS.

1. INTRODUCTION

Nowadays, researches on air vehicles are rapidly increasing owing to requirements and technological advancements. Acceptable new designs enable different types of aircrafts to have effective vertical take-off and landing. One of the well-known vehicles of this type is the conventional helicopter with hovering, high maneuverability and flying at very low speeds. Thus, autonomous or non-autonomous helicopters have attracted the attention of many researchers in military and civil fields. However, developing control algorithms for these vehicles have been considered as one of challenging problems of recent researches[1].

The flight control problem involves many complications. Altitude, payload, weather conditions, complex mechanisms, nonlinear aerodynamics and changes in flight conditions may disturb flight stability [2, 3]. The control algorithm for the aerial vehicles should overcome these difficulties. Thanks to Sliding Mode Controller (SMC) structure having strong ability to deal with high nonlinearity and
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2. PROBLEM STATEMENT AND PRELIMINARIES

The importance of real time application of the smooth and precise control of the prototype helicopter system is the main motivation of the paper. In the literature survey, authors encountered several SMC methods for TRMS. Thus, enhancement of the SMC for aerial vehicle will be appreciated. Some recent studies propose fractional order SMC design methods. However, this study employs SDD in the SMC that is simple to implement for multi-variable real time running system, yet effective for external disturbance suppression, capability of fast, precise and overshot-free control.

Recent improvements in implementation of FOD enable researchers to apply fractional order expressions to various research areas. We investigated that deviations from integer order differential terms of SMC structure affects the performance of control. Positive effect of the deviation motivates us to design more effective SMC structure for real time running prototype helicopter system.

2.1. Fractional Order Differentiation

Fractional-order derivative was discussed in 1690s with the letter between LeHopital and Leibniz [17]. Theoretical improvement of fractional calculus and related developments in computing, make it easy to use in different science and engineering fields [18], [19]. Fractional calculus is the general form of integer order integro-differential expressions and it is represented by $\int_{a}^{b} (d\tau)^{-\alpha}$. While $\frac{d^\alpha}{dt^\alpha}$ represents fractional derivative, $\int_{a}^{b} (d\tau)^{-\alpha}$ shows fractional order integration, where $\alpha$ is the order of integro-differential expression [20], [21]. In fact, the order of integro-differential expression, $\alpha$, defines an amount of deviation from integer order derivative or integration. So, one may say that the fractional order $\alpha$ is amount of deviation from the integer order one.

This paper discusses small deviations in derivative that make fine tuning on the effect of the derivation. Results are verified via simulation and real time applications. In other words, small deviations of the integer order derivative and integral make the effect of differentiation softer or harder depending on the sign of deviation $\alpha$. In this study, authors aimed to make fine tuning on the effect of differentiation in the SMC by using small deviations in the integro-differential expressions.
2.2. Conventional SMC

Conventional SMC is widely used in nonlinear systems to satisfy the stability when parameter fluctuations exist [22]. The SMC structure has two steps. In the first step, the sliding surface \( S(t) \) is selected. Secondly, the equivalent control law is determined for reaching and keeping on sliding axis. The \( S(t) \) is given as [22],

\[
S(t) = \left( \frac{d}{dt} + \lambda \right)^{n-1} e(t)
\]  

(1)

here, \( e(t) = x_d(t) - x(t) \) is tracking error of \( x(t) \), \( x_d(t) \) is reference input, \( \lambda \) is slope of the sliding surface, \( n \) is system order. The purpose of the control is to preserve the state variables on the sliding axis in (1). Following Lyapunov function was defined to determine the control law,

\[
V(S) = \frac{1}{2} S^2 (t)
\]  

(2)

If the Lyapunov function in (2) satisfies (3) for positive constant \( \eta \), the system reaches to \( S(t) = 0 \).

\[
\frac{1}{2} \frac{d}{dt} S^2 (t) \leq -\eta |S(t)|
\]  

(3)

One can obtain (4) using (3), to enforce \( e(t) \) to become zero all times.

\[
S(t)DS(t) \leq 0
\]  

(4)

where \( D \) is integer order derivative operator. The total control law \( u(t) \) consist of equivalent and discontinuous control terms as,

\[
u(t) = u_{eq}(t) + u_{d}(t)
\]  

(5)

This paper uses small deviations in the differentiation of the discontinuous and continuous parts of the control law.

2.3. SMC with FOD

Authors have recently proposed to use fractional order differentiation to improve the control performance of SMC for unstable time delay systems. SMC with FOD can be summarized from [11] as following:

The \( S(t) \) in (1) can be written as,

\[
S(\bar{\xi};t) = \left[ \sum_{k=0}^{i} \binom{i}{k} [D^\alpha]^k \lambda^{i-k} \right] e(t)
\]  

(6)

here \( i = n-1, \ k = 0,1, \ldots, n-1, \ (d/dt) = D \). The fractional operator \( D^\alpha \) widen the differentiation effects in (6) as [11],

\[
S(\bar{\xi};t) = \left[ \sum_{k=0}^{i} \binom{i}{k} [(D^\alpha)^k D^{\alpha_n}] \lambda^{i-k} \right] e(t)
\]  

(7)

where \( 0 < \alpha < 1 \). Equations (1)-(7) can be re-arranged for multi-variable systems to strengthen or to weaken the effect of the differentiation at sliding surface of SMC to make precise and overshoot-free control of MIMO systems. The proposed structure will also be robust against external disturbances. Next section provides the proposed controller design.

3. SMC-SDD DESIGN FOR LTI SYSTEMS

Consider the integer-order controllable system as,

\[
Dx(t) = Ax(t) + Bu(t) + d(x(t),t)
\]  

(8)

here, \( x(t), u(t) \) represents state vector and control input respectively. \( A \) and \( B \) are system parameter matrix and \( d(x(t),t) \) is an external disturbance.

Assumption 1: The disturbance with known upper bound is continuous derivable as, [22].

\[
\| d(x(t),t) \| < d_{\alpha} (x(t),t)
\]  

(9)

The sliding surface can be written for the LTI system as [22],

\[
S(t) = c^T x(t)
\]  

(10)

where \( c \in \mathbb{R}^{m \times n} \), \( m \times n \) represents parameter matrix. For \( DS(t) = 0 \) one can write the equivalent control law as,

\[
c^T Dx = c^T (Ax(t) + Bu(t))
\]  

(11)

\[
u_{eq}(t) = -(c^T B)^{-1} c^T (Ax(t))
\]  

(12)

One can write the total control law by adding discontinuous term, \( u_d(t) = -K \text{sgn} (S(t)) \), as,
\[ u(t) = - (c^T B)^{-1} c^T (A x(t) - K sgn(S(t))) \]  

**Assumption 2:** In accordance with (10) and (13), sliding mode exists if the matrix \((c^T B)\) has an inverse and \(\text{rank}(B) = m\) [22].

Let Assumptions 1 and 2 are satisfied under the control law (13). The sliding mode exists and \(e(t)\) will be zero at all times if (4) is fulfilled. One can derive the Lyapunov function as,

\[
DV = S(t) D S(t) \\
= S(t) \left[ c^T A x(t) + c^T B u(t) \right] \\
= S(t) \left[ c^T A x(t) + c^T B \left( - (c^T B)^{-1} c^T (A x(t)) - K sgn(S(t)) \right) \right] \\
\leq -K |S(t)|
\]

where, \(sgn(S(t)) = |S(t)|/S(t)\). It can be seen that, the tracking error converges to sliding surface if the parameter of \(K\) is chosen as \(K > 0\).

For the LTI system with \(n = 2\), the control law can be defined as,

\[
u(t) = - \begin{bmatrix} c_1 & c_2 \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}^{-1} \begin{bmatrix} c_1 & c_2 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix} \\
-K \text{sgn} \begin{bmatrix} c_1 & c_2 \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix} \]

Following equation defines the dynamic relations of the system in (15).

\[ X_2(t) = D X_1(t) \]  

Then, one can define following Remarks,

**Remark 1:** \(D^{1+\beta}\) weaken or strengthen the effect of differentiation in the continuity term, which forces \(DS(t)\) to be zero and keeps it to stay on the sliding surface, with \(X_2(t) = D^{1+\beta} X_1(t)\), where \(0 < \beta < 1\).

**Remark 2:** \(D^{\alpha}\) weaken or strengthen the effect of differentiation in the discontinuity term, which compensates the fluctuations while reaching to the sliding surface, with \(X_2(t) = D^{1+\alpha} X_1(t)\), where \(0 < \alpha < 1\).

### 3.1. Stability Analysis and Reachability Condition of SMC-SDD

Stability of dynamical systems is an essential property to design a necessary and satisfactory controller. There are several methods in the literature to investigate the stability of the integer systems. An integer-order LTI system is stable if all roots of the characteristic polynomial are in the left half of the \(s\) plane. However, the numerical solutions such as Routh-Hurwitz criteria cannot be used directly for a fractional order systems [23]. Oustaloup et. al was used Linear Matrix Inequality methods to check if matrix of Eigen values belong to a definite region in the complex plane [24]. This technique has been developed for ordinary and interval fractional order LTI systems. In addition, Matignon was investigated internal and external stabilities with the closed angular segment for linear fractional systems [25].

Fractional order LTI system in (17) can be considered in state-space form as [26],

\[
^0D_x^\alpha x(t) = Ax(t) + Bu(t) \\
y(t) = Cx(t)
\]

where \(y \in R^n\), \(u \in R^m\) are output, input and state vectors respectively. \(C \in R^{n \times m}\), \(B \in R^{n \times r}\), \(A \in R^{n \times n}\) and \(x = [\alpha_1, \alpha_2, \ldots, \alpha_n]^T\) are the fractional orders.

**Theorem 3.1:** If the condition in (18) is satisfied for \(\alpha_1 = \alpha_2 = \ldots = \alpha_n = \alpha\), one can say that the commensurate-order system in (17) is stable [26].

\[
|\text{Arg}(\text{eig}(A))| > \alpha \frac{\pi}{2}
\]

where \(\text{eig}(A)\) is eigen values and \(0 < \alpha < 2\).

However, Theorem 3.1 cannot be used if the system in (17) has various derivative orders. The system in (17) is called as an incommensurate-order system if \(\alpha_1 \neq \alpha_2 \neq \ldots \neq \alpha_n\). Let us define Fractional Order Differential Equation (FODE) with \(n\)-term as,

\[
a_1 D_0^{\alpha_1} y(t) + \ldots + a_n D_0^{\alpha_n} y(t) + a_n D_0^{\alpha_n} y(t) = 0
\]

here \(a_k \in R^n\) are real numbers \((\alpha_1 > \alpha_2 > \ldots > \alpha_n \geq 0\), \(a_k \in R^n\) are constant coefficients. The analytical solution of the
equation in (19) is assumed to be written as [27],

$$y(t) = \frac{1}{a_n} \sum_{n=0}^{\infty} \sum_{k_0, k_1, \ldots, k_{n-1}} (m; k_0, k_1, \ldots, k_{n-1}) \times$$
$$\prod_{i=0}^{n-1} \left( a_i \right) f_{\alpha_i}(t_a - a_{i+1} - a_i + \sum_{j=0}^{i} (a_{i+1} - a_i) k_{i+1} + 1)$$

(20)

where \((m; k_0, k_1, \ldots, k_{n-1})\) are coefficients and \(f_{\alpha_i}(t_a)\) is Mittag-Leffler function [27]. So the analytical solution of the FODE in (20) for \(\alpha_1 \neq \alpha_2 \neq \ldots \neq \alpha_n\) and \(u(t) = 0\) is stable if \(\lim_{t \to \infty} y(t) = 0\). Also, the stability test techniques for general fractional order LTI system can be found in [28]. Let the desired sliding mode surface is given by the fractional derivative form for LTI system in (17) as,

$$S(t) = c_1 X_1(t) + c_2 D^{\alpha} X_1(t)$$

(21)

**Remark 3:** The operator \(D^{\alpha}\) is the \((\pm \alpha - 1)^{th}\) order integration of \(X_1(t)\) that behave as a low-pass filter. Also, it may compensate the high-frequency fluctuations of disturbances. In this regard, the fractional-order sliding surface defined in (21) is smoother compared with conventional one.

Equation 15 can be rewritten using Remarks 1 and 2 as,

$$u(t) = -\left[ \begin{bmatrix} c_1 & c_2 \\ b_{11} & b_{12} \end{bmatrix} \right]^{-1} \left[ \begin{bmatrix} c_1 & c_2 \\ a_1 & a_2 \end{bmatrix} D^{i \beta} X_1(t) \right]$$

$$-K \text{sgn} \left[ \begin{bmatrix} c_1 & c_2 \\ D^{i \beta} X_1(t) \end{bmatrix} \right]$$

(22)

If (21) is rearranged,

$$c_1 X_1(t) + c_2 D^{\alpha} X_1(t) = 0$$

$$D^{\alpha} X_1(t) = -c_1 / c_2 X_1(t)$$

(23)

where \(0 < 1 \pm \alpha < 2\). It is shown according Theorem 3.1 that the sliding surface dynamics are asymptotically stable if the sliding plane constant, \(c_1 / c_2\), is selected to be positive.

Let us define the sliding surface according to (7) for higher order system \((n = 3)\) to show the same procedure as,

$$S(t) = \left[ \begin{bmatrix} \frac{2}{2} \left( D^D D^\infty \right) \lambda^0 \right] e(t) + \left[ \begin{bmatrix} \frac{2}{1} \left( D^D D^\infty \right) \lambda^1 \right] e(t)$$

$$+ \left[ \begin{bmatrix} \frac{2}{0} \left( D^D D^\infty \right) \lambda^2 \right] e(t)$$

$$= D^{\alpha_0} e(t) + 2D^{\alpha_0} \lambda e(t) + D^{\alpha_0} \lambda^2 e(t)$$

(24)

The curve of tracking error attains to \(S(t) = 0\), to satisfy the reaching law as,

$$D^{\alpha_0} e(t) + 2D^{\alpha_0} \lambda e(t) + D^{\alpha_0} \lambda^2 e(t) = 0$$

(25)

It can be seen from (25) that the expression of the sliding surface corresponds with the FODE in (19). If fractional orders are \(2 \pm \alpha_2 \neq 1 \pm \alpha_1 \neq 2 \pm \alpha_0\) and \(2 \pm \alpha_2 > 1 \pm \alpha_1 \geq 2 \pm \alpha_0\) then, the \(S(t)\) becomes incommensurate-order system. The analytical solution of this equation can be computed by using (20) satisfying the convergence of error function \(\lim_{t \to \infty} e(t) = 0\). Otherwise, the least common multiple (LCM) and Riemann sheets will be helpful to determine the stability of the incommensurate-order systems.

**Theorem 3.2** : Reaching mode can be shown by direct switching function approach that the system trajectory is under the following reaching condition [29].

$$\lim_{t \to \alpha^0} S_i(t) < 0 \text{ and } \lim_{t \to \alpha^0} DS_i(t) > 0$$

(26)

$$S_i(t)DS_i(t) < 0 \quad i = 1, \ldots, m$$

(27)

One can add small differentiation deviations to the time derivative of sliding surface in (11) as,

$$DS(t) = \left[ \begin{bmatrix} c_1 & c_2 & c_3 \\ a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \right] \begin{bmatrix} X_1(t) \\ X_2(t) \\ X_3(t) \end{bmatrix}$$

(28)

where, \(X_2(t) = D^{i \beta} X_1(t)\). The dynamics of the switching function can be specified by using Gao’s constant rate reaching law approach as in [30].

$$DS(t) = -K_d \text{ sgn} \left( S(t) \right)$$

(29)

where \(K_d > 0\). By using (28) and (29),
According to Theorem 3.2, \( S(t)DS(t) \leq 0 \) and \( e(t) \) is zero at all times and the system is stable in the presence of the SMC-SDD controller with the time derivative of saturation function.

4. EXPERIMENTAL SETUP

The flight control simulator, TRMS, is composed of two propellers joined by a shaft pivoted on its base. The shaft can rotate freely on both vertical and horizontal planes. TRMS is widely used in laboratories in the literature to mimic the flight controls of a helicopter.

TRMS control involves some difficulties, due to the mutual interaction between the two axes, and the nonlinearity in motion of mechanism [31]. The control purpose of TRMS is to achieve a quick and accurate position. A mathematical model of the TRMS should be correctly derived to design SMC-SDD accurately. Approximate mathematical models of the TRMS are obtained by using Newtonian and Lagrangian methods by considering effects of various forces existing in the system[32].

4.1. Dynamical Model of TRMS

Basic definitions in (35)-(42) represent vertical and horizontal model of the system. These equations are obtained by considering the system dynamics. Dynamic model of the vertical subsystem is derived as follows [5], [6], [30],

\[
\frac{d\psi(t)}{dt} = D\psi(t) \tag{35}
\]

\[
D^2\psi(t) = \frac{a}{I_1} \tau_1(t)^2 + \frac{b}{I_1} \tau_1(t) - \frac{M_e}{I_1} \sin\psi(t) + 0.0326 \sin(2\psi(t)) \left(\frac{D\varphi(t)}{I_1}\right)^2 - \frac{B_{we}}{I_1} D\psi(t) \tag{36}
\]

\[
-k_{\psi_1} \cos\psi(t) \left(\frac{a_1 \tau_1(t)^2 + b_1 \tau_1(t)}{I_1}\right) D\varphi(t) \tag{37}
\]

Dynamic model of the horizontal subsystem can be given as,

\[
\frac{d\varphi(t)}{dt} = D\varphi(t) \tag{38}
\]
\begin{equation}
D^2 \varphi(t) = \frac{a_2}{I_2} \tau_2(t)^2 + \frac{b_2}{I_2} \tau_2(t) - \frac{B_w}{I_2} D\varphi(t) - \frac{1.75}{I_2} k_v \left( a_1 \tau_1(t)^2 + b_1 \tau_1(t) \right)
\end{equation}

\begin{equation}
D \tau_2(t) = -\frac{T_{20}}{T_{21}} \tau_2(t) + \frac{k_2}{T_{21}} u_\varphi(t)
\end{equation}

where \( \psi(t) \) and \( \varphi(t) \) are pitch and yaw angles, \( D\psi(t) \) and \( D\varphi(t) \) are angular velocity around the vertical and horizontal axis, \( \tau_1(t) \) and \( \tau_2(t) \) are output torque of main and tail rotors modeled as a first order linear system, \( u_\varphi(t) \) and \( u_\psi(t) \) are voltage applied to main and tail rotors respectively. Remaining parameters of the TRMS are provided in [30]. The mathematical model in (35)-(40) can be represented in state space form as,

\begin{equation}
D \begin{bmatrix} \psi(t) \\ \tau_1(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -B_w/I_1 & b_1/I_1 & \psi(t) \\ 0 & 0 & \tau_1(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \tau_1(t) \end{bmatrix} + \begin{bmatrix} \psi(t) \\ 0 \\ u_\varphi(t) + f_\varphi(t) \end{bmatrix}
\end{equation}

\begin{equation}
D \begin{bmatrix} \varphi(t) \\ \tau_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -B_w/I_2 & b_2/I_2 & \varphi(t) \\ 0 & 0 & \tau_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \tau_2(t) \end{bmatrix} + \begin{bmatrix} \varphi(t) \\ \tau_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ k_2/T_{21} u_\psi(t) + f_\psi(t) \end{bmatrix}
\end{equation}

where \( f_\varphi(t) \) and \( f_\psi(t) \) are assumed to be unknown nonlinearity.

\section{SMC-SDD DESIGN FOR TRMS}

In this section, the proposed SMC-SDD structure given in Section 3 is adapted to the TRMS to eliminate the undesirable effect of external disturbance and to obtain fast and overshoot-free step response. Approximate mathematical model of the system can be rewritten in the following form as a convenience,

\begin{equation}
X_1(t) = X_1(t) + B(t)
\end{equation}

\begin{equation}
X_2(t) = X_2(t) + \psi(t)
\end{equation}

\begin{equation}
X_3(t) = X_3(t) + \varphi(t)
\end{equation}

\begin{equation}
\begin{bmatrix} X_1(t) \\ X_2(t) \\ X_3(t) \end{bmatrix}^T = [\psi(t) \ \varphi(t) \ \tau_1(t)]^T
\end{equation}

where \( X(t) = [X_1(t) \ \ X_2(t) \ \ X_3(t)]^T \) is state vector and input \( u(t) = u_\psi(t) \) are defined in the case of pitch angle response, while \( X(t) = [X_1(t) \ \ X_2(t) \ \ X_3(t)]^T \) and input \( u(t) = u_\varphi(t) \) are defined for yaw angle. The \( \gamma \) parameters are defined as \( \gamma_w = -B_w/I_1 \), \( \gamma_w = b_1/I_1 \), \( \gamma_20 = -T_{20}/T_{21} \) and \( \gamma_{30} = -B_{30}/I_2 \), \( \gamma_w = b_2/I_2 \), \( \gamma_{30} = -T_{20}/T_{21} \) for the vertical and horizontal subsystem respectively. The error dynamics is given as,

\begin{equation}
e(t) = X(t) - X_d(t)
\end{equation}

where \( X(t) = [X_1(t) \ \ X_2(t) \ \ X_3(t)]^T \) is state vector, \( e(t) \) is tracking error of state variable \( X(t) \) and \( X_d(t) \) is desired input. The sliding surface can be written as follows,

\begin{equation}
S(t) = c^T e(t)
\end{equation}

where \( c = [c_1 \ c_2 \ c_3] \) is coefficient vector of the error and \( e(t) = [e_1(t) \ e_2(t) \ e_3(t)]^T \) is the error state vector. The sliding surface given in (44) can be rewritten as,

\begin{equation}
S(t) = c_1 e_1(t) + c_2 e_2(t) + c_3 e_3(t)
\end{equation}

where \( e_1(t) = X_1(t) - X_d(t) \), \( e_2(t) = X_2(t) \) and \( e_3(t) = X_3(t) \). Thus the sliding surface becomes as,

\begin{equation}
S(t) = c_1 (X_1(t) - X_d(t)) + c_2 DX_1(t) + c_3 X_3(t)
\end{equation}

\begin{equation}
DS(t) = c_1 (X_2(t) - DX_d(t)) + c_2 DX_2(t) + c_3 DX_3(t)
\end{equation}
Table 1. Parameters of the TRMS

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Definitions</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_w$</td>
<td>Friction momentum function parameter of vertical axis</td>
<td>$6 \times 10^{-3} \text{Nms/rad}$</td>
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<tr>
<td>$B_h$</td>
<td>Friction momentum function parameter of horizontal axis</td>
<td>$1 \times 10^{-5} \text{Nms/rad}$</td>
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<td>$I_1$</td>
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</tr>
<tr>
<td>$k_b$</td>
<td>Gain of cross reaction momentum</td>
<td>$-0.2 \text{ s/rad}$</td>
</tr>
<tr>
<td>$M_g$</td>
<td>Gravity momentum</td>
<td>0.32 m</td>
</tr>
</tbody>
</table>

Equation (49) is obtained by substituting (43) into (48) as,

$$ DS(t) = c_1 \left( X_1(t) - DX_1 \right) + c_2 \left( \gamma_1 X_2(t) + \gamma_2 X_3(t) \right) + c_3 \left( \gamma_3 X_3(t) + Bu(t) \right) $$

(49)

The derivative of the sliding surface can be written by using Gao’s reaching law as in [30],

$$ DS(t) = -K_d \text{sgn}(S(t)) $$

(50)

The total control law $u(t)$ is obtained by using (49) and (50) as follows,

$$ u(t) = \frac{1}{c_2 B} \left( -c_1 \left( X_1(t) - DX_1 \right) - c_2 \left( \gamma_1 X_2(t) + \gamma_2 X_3(t) \right) + c_3 \gamma_3 X_3(t) - K_d \text{sgn}(S(t)) \right) $$

(51)

One can write the control law relating to the pitch and yaw angle from (51), respectively as,

$$ \begin{align*}
\tau_1 &= \frac{k_1}{T_{12} + T_{20}} u_p \\
\tau_2 &= \frac{k_2}{T_{12} + T_{20}} u_p
\end{align*} $$

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\[
u_c(t) = -\left[ \begin{bmatrix} c_{\psi_1} & c_{\psi_2} & c_{\psi_3} \\ 0 & 0 & 0 \\ k/T_{1} \\ \end{bmatrix} \right]^{-1} \left[ \begin{bmatrix} 0 & 1 & 0 \\ 0 & -b_{\psi}/I_{1} & b_{\psi}/I_{1} \\ 0 & -T_{m}/T_{1} & 0 \\ \end{bmatrix} \right] \left[ \begin{bmatrix} \psi(t) \\ \dot{\psi}(t) \\ \tau_1(t) \\ \end{bmatrix} \right] - K_{d\psi} \text{sgn} \left[ \begin{bmatrix} c_{\psi_1} & c_{\psi_2} & c_{\psi_3} \\ \end{bmatrix} \right] \left[ \begin{bmatrix} \psi(t) - \psi_d(t) \\ \dot{\psi}(t) - \dot{\psi}_d(t) \\ \tau_1(t) - \tau_d(t) \\ \end{bmatrix} \right] \right] (54)
\]

\[
u_c(t) = -\left[ \begin{bmatrix} c_{\psi_1} & c_{\psi_2} & c_{\psi_3} \\ 0 & 0 & 0 \\ k_{z}/T_{21} \\ \end{bmatrix} \right]^{-1} \left[ \begin{bmatrix} 0 & 1 & 0 \\ 0 & -b_{\psi}/I_{2} & b_{\psi}/I_{2} \\ 0 & -T_{m}/T_{21} & 0 \\ \end{bmatrix} \right] \left[ \begin{bmatrix} \varphi(t) \\ \dot{\varphi}(t) \\ \tau_2(t) \\ \end{bmatrix} \right] - K_{d\varphi} \text{sgn} \left[ \begin{bmatrix} c_{\psi_1} & c_{\psi_2} & c_{\psi_3} \\ \end{bmatrix} \right] \left[ \begin{bmatrix} \varphi(t) - \varphi_d(t) \\ \dot{\varphi}(t) - \dot{\varphi}_d(t) \\ \tau_2(t) - \tau_d(t) \\ \end{bmatrix} \right] \right] (55)
\]

A block diagram of SMC-SDD control structure is given in Fig. 1, using (54) and (55). The sgn function is changed with the derivative form of saturation function in continuous time to overcome the chattering problem. This function can be written as follows,

\[
\frac{S(t)}{\left| S(t) \right| + \delta} \tag{56}
\]

where \( \delta > 0 \) is provided as a small value.

![Block diagram of proposed SMC-SDD](image1.png)

**Figure 1.** Block diagram of proposed SMC-SDD.

### 6. SIMULATION AND EXPERIMENTAL RESULTS

This section provides simulation and experimental results for vertical and horizontal axes of the TRMS. The SMC-SDD structure, proposed in this paper is implemented on the TRMS to demonstrate the performance of control effort. An external disturbance is applied to the system to show the robustness of the proposed control strategy. The hardware setup of the TRMS has two encoders for position feedback of the propellers and I/O cards for data acquisitions. The Matlab/Simulink modules are used to obtain the model of the TRMS and SMC-SDD controller. For simulation and experimental studies, sampling frequency was selected as 1 MHz. The continuous fraction expansion method was used to obtain Forth order rational approximations of the FOD operator to simulate SDD [33]. Block diagram of the closed-loop system and experimental setup of TRMS are given in Figs. 2 and 3 respectively.

The initial conditions of the TRMS were considered as \( \psi(0) = \varphi(0) = 0 \) and the desired values of the pitch and yaw angles were chosen as \( \psi_d(t) = 0.5 \), \( \varphi_d(t) = 0.5 \). The SMC-SDD parameters for the vertical and horizontal subsystem were given as \( K_{d\psi} = 3 \), \( K_{d\varphi} = 2.4 \), \( \delta_\psi = 0.57 \), \( \delta_\varphi = 0.8 \), \( c_{\psi} = [5 0.5 0.01] \), \( c_{\varphi} = [4 0.1 0.2] \).

![Block diagram of the controlled system](image2.png)

**Figure 2.** Block diagram of the controlled system.

A low pass filter was used at the feedback line to reject noise of the encoder. The parameters of CSMC and SMC-SDD control structures are the same except the fractional orders \( \alpha \) and \( \beta \) in SMC-SDD. In other words, One can obtain CSMC from SMC-SDD if \( \alpha = 0 \) and \( \beta = 0 \).
Different values of differentiation orders $\alpha$ and $\beta$ in (54) and (55) are used in block diagram of SMC-SDD in Fig. 1 to see the impacts of the SDD on the system performance. Figs. 4 and 5 present the simulation and experimental results of the pitch angle response of TRMS respectively, to illustrate the effect of parameter $\alpha$ while the $\beta$ parameter was zero. One can see from inset figures that when an external disturbance is applied to the system, the amplitude of the disturbance by the SMC-SDD with $\alpha = 0.2$ parameter is smaller than the CSMC. The disturbance $d(t) = 4V$ has continued for time interval 20-21 sec. The numerical simulation in Fig. 4 confirms the experimental results in Fig. 5 in tracking desired angles and suppression the disturbance. Figs. 6 and 7 show the simulation and experimental results of the performance of the CSMC and SMC-SDD, with the $\alpha = 0$ and $\beta = (0.1, 0.2, 0.3)$. The disturbance remains same as in the previous case. One can see from inset figures that when an external disturbance is applied to the system, the amplitude of the disturbance by the SMC-SDD method is smaller than that of the CSMC. The $\beta$ is more effective to suppress the disturbances than the $\alpha$.

The higher values of $\beta$ parameter usually compensate disturbance but causes higher overshoots. Simulation and experimental results in Figs. 8 and 9 were given to demonstrate the effect of $\alpha$ and $\beta$ parameters together on SMC-SDD. The disturbance was added to the system as in previous two cases. The desired response to SMC-SDD was achieved at values $\alpha = -0.4$ and $\beta = 0.25$. There is no doubt that the variation of $\alpha$ and $\beta$ parameters together is sufficient to improve the response of the system in the presence of disturbance.
Despite the high disturbance suppression ability, the parameter $\beta$ causes some overshoot. However this overshoot can be eliminated by using $\alpha$ and $\beta$ together. Also, $\beta$ enhances the disturbance suppression effect of $\alpha$ as in pitch angle response of TRMS. It is shown that the proposed controller method with $\alpha = -0.3$ and $\beta = 0.1$ parameters is more robust than the CSMC method for yaw angle response.

$$d(t) = \begin{cases} 2V, & 20 < t < 21 \\ -2V, & 21 < t < 22 \end{cases}$$ (57)
7. CONCLUSIONS

In this paper, CSMC structure was modified to enhance the control success of the multivariable TRMS by adding SDD. The SMC-SDD and CSMC method were used for the vertical and horizontal rotation of TRMS to decrease the steady state error. Numerical simulations confirm the experimental results in tracking desired angles and suppression of the disturbance. The SDD at the differential term of SMC strengthen or weaken the effect of differentiation according to the sign of deviation. The proposed method enables user to make fine-tuning for better output response. SMC-SDD successfully suppresses the external disturbances and exhibits fast and overshoot-free step response. The robustness of the system for the proposed method is better than the CSMC with the small tracking errors.

8. REFERENCES


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