 COMPUTATIONALLY EFFICIENT ASSESSMENT OF FIGHTER AIRCRAFT MISSION SURVIVABILITY WITH PROBABILISTIC GRAPHICAL MODELS

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ABSTRACT
This paper proposes a probabilistic model for assessment of fighter aircraft mission survivability. Mission survivability analysis is a critical phase for both design of the fighter aircraft and evaluation of its performance. The standard deterministic performance metrics such as aircraft agility and manoeuvrability are not sufficient to measure mission survivability, since the probability of aircraft surviving the missions depends heavily on lethality and position of threats, such as surface to air missile systems. Since the dynamics and parameters of the threats are mostly uncertain, previous works proposed several different probabilistic models for modelling the mission survivability of fighter aircraft under uncertain threats. However, most of the existing models either oversimplify the problem or leads to in complicated high dimensional probability distributions, which are unfeasible for evaluation of large-scale missions. In this study, we fuse the threat and sensor models from several existing works and show that the mission survivability can be modelled as a probabilistic graphical model, which enables rapid sampling and Monte Carlo evaluation of survivability, even on large-scale missions that involve many different threat and sensor networks. In addition, we show that graphical representation can also be used for addressing the inverse problem of determining required aircraft performance parameters for a specified survivability rate.

Keywords: Fighter Aircraft Survivability, Missions Survivability, Probabilistic Threat Models, Probabilistic Graphical Models.

SAVAŞ UÇAĞI GÖREV HAYATTA KALABİLİRLİĞİNİN OLAŞILIKSAL GRAFIKSEL MODELLER İLE HESAPSAL OLARAK VERİMLİ ANALİZİ

ÖZET

Anahtar Kelimeler: Savaş Uçakları, Görev Hayatta Kalabilirliği, Olaşılsal Tehdit Modelleri, Olaşılsal Grafiksel Modeller.
1. INTRODUCTION

Since the early days of fighter aircraft design, survivability was seen as a critical parameter for assessing fighter performance [1]. Initially survivability was evaluated as a function of the maneuverability and agility of the aircraft, which depends on aerodynamics characteristics and engine properties [2]. However, as the field progressed, it was shown that the models of sensor networks and threat lethality properties also have a huge effect on survivability of the fighter. Hence the modern survivability analysis includes both the analysis of the fighter aircraft capacity to evade threats, as well as the sensing and killing capabilities of the threat [3]. Hence, simulating the aircraft performance under threats is of utmost importance for assessing mission survivability. However, due to nature of military missions, the knowledge of threat dynamics, position and capabilities are largely uncertain beforehand. Hence the simulators and tools for survivability analysis need to handle these uncertainties for realistic survivability computations.

The main objective of this work is to present a computationally efficient probabilistic framework for assessing fighter aircraft mission survivability. This objective is achieved via exploiting the probabilistic independencies between aircraft parameters, sensor network properties and lethality capabilities of threats by modeling the associated distributions via probabilistic graphical models (PGMs) [4]. PGMs represent independency of random variables with graphs, which enables rapid inference and probability computations by exploiting the topology. We show that PGMs and Markov Chains developed in this work enable both rapid assessment of mission survivability and also computation of critical probabilities, such as probability of being detected by a certain sensor or the probability of being hit by a certain threat.

1.1 Previous Work

Most of the work on mission survivability originated from path planning and motion planning research, where the researchers from computer science and robotics attempted to extend the classical trajectory management algorithms to include threat models. Some of these works model threats simply as restricted no fly zones [5,6], whereas others developed algorithms that minimize the time spent in areas that are under the influence of threats [7,8]. More advanced route planning algorithms that take sensor noise and enemy maneuver uncertainty into account were also studied [9,10].

On the other hand, there are also a significant amount of work done on developing realistic threat and sensor models. Such models usually model the probability of aircraft being hit as a function of weapon properties and configuration of the aircraft [11,12]. There are also probabilistic radar models that give the probability of detection based on radar equations and aircraft stealth properties [13].

One of the most complete works done on mission survivability analysis was conducted by Erlandsson and Niklasson [14]. In their work, authors developed a Markov Chain based survivability model that takes alternative flying routes as input and calculates the survivability rate depending on the sensor and weapon locations in the enemy zone. The model also takes into account that sensors can share information, and detection and hit events are modelled as random variables. Although this model offers one of the most advanced mission survivability computations, it does not take advanced weapon and sensor models into account (all the events are binary random variables) and it also does not include the effect of varying aircraft parameters, such as aerodynamics and engine performance.

It is seen that although a spectrum of different tools exists for assessing mission survivability, best to author’s knowledge there does not exist a single work that takes both aircraft parameters, and the dynamics of sensor-threat networks into account. This is partially due to fact that as the variables in a probabilistic models increase, sampling from these models and performing inference on them get computationally infeasible. Although there were attempts to utilize graphical methods for pilot decision systems [15], a large-scale mission wide analysis was not conducted.

Contributions of our work is summarized in the following bullets:

- In this work, we present the interactions between aircraft, sensors and weapons in terms of graphical models, which enables fast sampling and inference for survivability analysis.
- We also show that using graphical models enable inverse analysis of survivability. By using graphical inference methods, such as Markov Chain Monte Carlo, it is possible to infer an interval of aircraft performance characteristics that would result in specified survivability rate.

2. BACKGROUND

This section gives the necessary background on probabilistic graphical models (PGMs), Markov Chains and related sampling and inference algorithms. The interested reader is referred to textbooks for a deeper analysis of these subjects [16].

2.1 Probabilistic Graphical Models

Let \( X = (X_1, X_2, \ldots, X_n) \) be a collection of real valued discrete random variables. Let \( m \) be the number of distinct values each random variable \( X \),
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can take. Hence, to specify the full joint probability distribution \( P(X) = P(X_1, X_2, \ldots, X_n) \), one would require \( n^m \) parameters. Due to this exponential complexity, both sampling and inference on large-scale distributions are computationally challenging [16].

Fortunately, many probabilistic models in engineering and science enjoy some conditional independence properties, which allows one to neglect a large number of parameters in the joint probability model by factorizing it into smaller models. PGMs utilize conditional independencies by representing each random variable as vertices and the conditional independencies between variables as edges in a graph. In this work, we are interested in a particular form of graphical models known as Bayesian Networks [17], which uses directed acyclic graphs for factorizing probability distributions.

![Figure 1: An example of a graphical model.](image)

An exemplary graphical model (Bayesian Network) is given in Figure 1, where random variables are denoted by the letters \((A, B, C, D, E)\). The formula for factorization of the joint probability distribution based on the graphical model is given as follows:

\[
P(X) = P(X_1, X_2, \ldots, X_n) = \prod_{i=1}^{n} P(X_i | Pa(X_i))
\]

Where the function \( Pa(X_i) \) returns the parent variables of the node \( X_i \). In other words, each random variable is conditionally independent from the rest of the random variables given the value of their parent variables. For instance, the distribution given in Figure 1 factorizes as:

\[
P(A, B, C, D) = P(A) P(B) P(C|A, B) P(D|C, E) P(E)
\]

Note that if each variable in this example were a binary random variable, the joint probability distribution would need \( 2^5 = 32 \) parameters, on the other hand, thanks to factorization offered by the Bayesian network, only \( 2 + 2 + 8 + 8 = 20 \) parameters were needed for the factorized distribution. Although the difference is subtle in this toy example, for the large-scale scenarios with many independencies, graphical models can provide a huge number of savings for storing the distribution [4].

The graphical models also enable utilization of incredibly efficient sampling and inference algorithms. In general, sampling from a high dimensional joint distribution is a computationally expensive task [16]. On the other hand, sampling from a Bayesian Network is much cheaper, since the direction of the edges hint that sampling from the joint distribution is the same as sampling sequentially from a bunch of lower dimensional conditional probability distributions.

In many engineering problems, we want to use the probabilistic models for inference; which is computing probability distributions conditioned on the observed variables. Graphical models lead to very efficient inference algorithms, since the independency between variables can be exploited to speed up the probability estimation process. Both optimization based [18] and sampling based [19] inference algorithms for graphical models exist, in this work we will utilize sampling based algorithms for running inference on our survivability model.

### 2.2 Markov Chains

Let \( X_k \) be a discrete time random process where the time is indexed by variable \( k = 0, 1, \ldots \). We say that \( X_k \) is a Markov Chain, if the probability distribution factorizes according to:

\[
P(X_{k+1}, X_k, X_{k-1}, \ldots, X_0) = P(X_{k+1} | X_k)
\]

In other words, current state evolves according to the state at previous time step and it is independent of all the previous states. Let \( X_k \) take \( n \) distinct values. A Markov Chain is usually parameterized by a state transition matrix \( \Lambda \in [0,1]^n \), where \( \lambda_{ij} \) represents the probability \( P(X_{k+1} = j | X_k = i) \).

### 3. The Survivability Model

In this section we present the details of our mission survivability model. The model consists of a single fighter aircraft flying through a network of enemy sensors (radars) and weapons (surface to air missiles). The overall interaction between different components of the model at a fixed time \( k \) is given in Figure 2.
3. Aircraft-Sensor Interaction Model

This model is based on [13]. We assume that there are $n_{sens}$ number of sensors (radars) in the mission area. For each radar, the probability of detecting the aircraft is described via the graphical model given in Figure 3.

\[
P(A/C \text{ is detected by radar } i | A/C \text{ and Radar properties}) = \frac{1}{1 + \left( \frac{c_1 R^2}{\sigma} \right)^{c_2}}
\]

where $c_1, c_2$ are radar specific constants that represent the signal power and processing capabilities, $R$ is slant range between the radar and aircraft, and finally $\sigma$ is the RCS of the aircraft. Note that the model allows radars with different capabilities, each radar in the network might have different power etc.

Also note that aircraft and radar properties were also defined as random variables in this model. This allows greater flexibility in the analysis, since there might be some uncertainty associated with radar and aircraft properties. For instance, if the exact location of the radar or signal power is unknown, this uncertainty can be reflected in the model. Similarly, if the fighter aircraft is still in design process and final geometry is not determined, the model allows expression of this uncertainty as a random variable. Due to continuous nature of these random variables, they are modelled as independent normal distributions, unlike detection random variable, which is binary.

Also note that the described model only gives the probability of being detected by a single radar. We assume that $n_{sens}$ radars work independently, and aircraft is detected whenever at least one sensor detects it. Hence the probability of detection is:

\[
P(A/C \text{ is detected}) = P \left( \sum_{i=1}^{n_{sens}} D_i \geq 1 \right)
\]

Where $D_i = 1$ if the aircraft is detected by radar $i$ and $D_i = 0$ otherwise.

3.2 Sensor-Weapon Network Model

After the aircraft is detected, we assume that sensor network alerts the weapon network to fire at the aircraft. We assume that there are $n_{weap}$ number of enemy weapons in the mission environment and the sensors and weapons are connected to each other via a communication network. An example model of the network is presented in Figure 4. The communication network is modelled as a bipartite graph, where the $n_{sens} \times n_{weap}$ adjacency matrix is given by $C$. Elements $c_{ij}, i = 1, \ldots, n_{sens}, j = 1, \ldots, n_{weap}$ describe the links in the network. $c_{ij} = 1$ denotes that sensor $i$ can communicate with weapon $j$, otherwise $c_{ij} = 0$. A weapon can fire at the aircraft if and only if it is alerted by a sensor in its network. Correspondingly, a sensor alerts the weapons in its network only if it detects the aircraft.
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3.3 Aircraft-Weapon Interaction Model
The graphical model is presented in Figure 5. The probability of a weapon killing the aircraft depends on aircraft’s agility metrics, the relative position between aircraft and the weapon and the weapon (SAM) properties.

As reviewed in the Introduction, several equations for probability for modelling probability of kill exists, however none of them were deemed to be sufficiently detailed for the analysis in this paper. For this purpose, we set the axial agility as the thrust/weight ratio, lateral agility as time to capture the 90-degrees roll angle \( \tau_{90} \)[20]. Then we have used a 6 DOF nonlinear F-16 model [21] and a 3DOF SAM interceptor model [22] to conduct a large scale Monte Carlo analysis for different intervals of agility metrics and SAM properties, and tabulated the resulting probability distribution:

\[
P(A/C \text{ is killed by weapon } j | A/C \text{ and SAM properties}) .
\]

Note that, similar to how sensor-aircraft interaction was described, all the aircraft agility parameters and weapon parameters are also modelled as continuous random variables with normal distribution. Naturally, aircraft is assumed to be killed when at least one weapon hits it:

\[
P(A/C \text{ is killed}) = P\left( \sum_{j=1}^{n_{\text{sw}}} F_j \geq 1 \right),
\]

where \( F_j = 1 \) if the weapon \( j \) kills the aircraft and \( F_j = 0 \) otherwise.

3.4 Mission Markov Chain Model and Survivability Calculation
The model given in Figure 1, allows simulating aircraft being detected or hit at a specific time instant \( k \), however since the aircraft is flying through the mission environment, its position is updated at every time instant, which effects the probability calculation at each time step, since the probability of detection and being hit are strong correlated with the relative position between aircraft and sensor/weapon network. To model the changing trajectory of the aircraft, we assume that it is flying through a pre-specified sequence of waypoints \( WP_i, i=1,..,n_{wp} \), and let \( X_k \) represent the waypoint aircraft is present at time \( k \). Since the next location of the aircraft only depends on the previous location, the whole mission can be represents as a Markov Chain (see Section 2.2), similar to how it is done in Ref. [14]. An example scenario with a sequence of waypoints, radars and weapons are displayed in Figure 6.

We assume that the scenario has a limited time horizon given by \( n_{hor} \). The probability of mission survival is defined as aircraft being alive by the end of the mission. Hence mission survivability rate \( P_s \) is computed as:

\[
P_s = 1 - \bigcup_{k=1}^{n_{hor}} P(A/C \text{ is hit at time } k).
\]
4. SIMULATION RESULTS
In this section we present our Monte Carlo simulation results across different scenarios. Since the model presented in section 3 involves many different parameters (such as radar and weapon parameters, connectivity of the weapon-sensor network, number of waypoints etc.), we decided to fix only some of the critical parameters, such as the mission area size, number of weapons and sensors, and set the rest of the parameters randomly from a specified intervals and average the results across the samples.

4.1 Demonstration of Model Capability
First we assess if the presented model can capture the probability of mission survival as a function of mission difficulty. We investigate 6 different scenarios with varying number of sensors, weapons and weapon-sensor network connectivity. Across all scenarios, the model of the fighter aircraft ([21]), number of waypoints (set to 10) and size of the mission area (150 x 150 km) was held fixed, rest of the values were sampled randomly from intervals collected from various sources [2,3,13,20]. All results were averaged over 10000 samples obtained from sampling the probability distributions given by the graphical models. Results are displayed in Table 1.

Examining Table 1 reveals how the survivability rate is affected by mission difficulty. In the first 3 scenarios, the average connectivity between sensors and weapons are held low, on average, each sensor is only connected to one weapon, hence the probability of firing a weapon is relatively low. We see that aircraft maintains over 90% survivability rate in the first scenario. As the number of weapons and sensors increases, survivability starts falling down and hits to almost 50% for the scenario with 50 sensors and 10 weapons. This is expected since increasing the number of sensors increases the detection rate, which in turn increases the probability of being hit by a missile. The last 3 scenarios uses the same number of sensors and weapons as the first 3 scenarios, however the average connectivity in the sensor-weapon network is increased to 2. That is, each sensor alerts 2 weapons on average. We see that survivability rate goes down much sharper compared to first 2 scenarios. The analysis shows that degree of connectivity in the sensor-weapon network can have a much larger impact on survivability rate compared to sheer number of total sensors and weapons in the area. To sum up, the analysis shows that the proposed model is capable of capturing mission survivability across scenarios with varying difficulty and scale.

4.2 Demonstration of Computational Efficiency
To demonstrate the computational efficiency of the model, we compare the average number of samples and CPU time required to estimate \( P \), within a certain tolerance, between the full joint probability distribution and the factorized model given by the graphical model of Section 3. Results are given in Table 2.

Table 2: Comparison of sample and time complexity between the full and factorized model

<table>
<thead>
<tr>
<th>( n_{sens} + n_{weap} )</th>
<th>#Samples</th>
<th>#Samples</th>
<th>CPU Time</th>
<th>CPU Time</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Joint Dist.</td>
<td>Graphical Model</td>
<td>Joint Dist.</td>
<td>Graphical Model</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>1210.2</td>
<td>429.3</td>
<td>67.4 sec</td>
</tr>
<tr>
<td>100</td>
<td>722,54</td>
<td>333,157</td>
<td>1,677,18</td>
<td>N/A</td>
</tr>
<tr>
<td>1000</td>
<td>N/A</td>
<td>32,544,2</td>
<td>4.6 hours</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Examining Table 2 reveals the significant advantage of using a factorized model over the full model. The number of samples and CPU time for both approaches are only comparable for small-scale models with 10 weapons and sensors. As the number of elements in the mission increases, the joint probability becomes infeasible to use, number of samples and CPU time blows up around 100 sensors and weapons, and from that point our computers were unable to generate meaningful results for large-scale scenarios. On the other hand, number of samples and CPU time scales up almost logarithmically for the graphical model, due to exploit of conditional independence between sensor and weapon models.

4.3 Demonstration of Inference Results
In our final demonstration, we use the Markov Chain Monte Carlo (MCMC) method [19] to address some inverse problems related to survivability assessment. In the first set of simulations, we fix the scenario to include only sensors and no weapons. We assume that the aircraft is detected at a specific time instant and try to find which sensor (radar) was responsible for the detection. The algorithm works as follows: in the graphical model depicted in Section 3, we accept ‘Aircraft Detected’ as an observed variable and run MCMC algorithm to compute probability distribution over which radars detected the aircraft. Then we simply take the mode of the distribution to estimate the responsible radar. Results across scenarios with different number of sensors are given in Table 3.
Table 3: MCMC estimation for estimating the sensor responsible for detection

<table>
<thead>
<tr>
<th>n_{sen}</th>
<th>Probability of Estimating the Responsible Sensor</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>%75.2</td>
</tr>
<tr>
<td>50</td>
<td>%50.2</td>
</tr>
<tr>
<td>100</td>
<td>%12.4</td>
</tr>
</tbody>
</table>

Table 3 shows that for low number of sensors the inference algorithm is moderately successful for estimating the sensor responsible for detection. As expected, as the number of sensors increase, the same probability gets lower, since the radar array starts to become denser and hence it becomes difficult to estimate which sensor was responsible for detection. Nevertheless, such analysis tools are very useful for evaluation of the missions, since they can enable assessment of which radars are the most critical in the sensor network.

For the final simulation, we specify $P_s = %95$ for the model and try to infer probability distributions on aircraft RCS and time to capture 90 degree roll angle $t_{90}$ required for achieving the specified $P_s$. Results across different number of weapon and sensor combinations is given in Table 4.

Table 4: Estimation of mean aircraft parameters required to capture %95 survivability rate

<table>
<thead>
<tr>
<th>n_{sen}</th>
<th>n_{weap}</th>
<th>Mean RCS (m$^3$)</th>
<th>Mean t_{90} (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>2</td>
<td>3.1</td>
<td>1.53</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>1.2</td>
<td>1.22</td>
</tr>
<tr>
<td>20</td>
<td>6</td>
<td>0.05</td>
<td>1.12</td>
</tr>
</tbody>
</table>

Table 4 hints that the stealth characteristics of the aircraft, specifically RCS, might be a more critical factor in aircraft survivability. Results show that the mean lateral agility was found to be close across different scenarios, whereas RCS was progressively lowered as the number of weapon and sensors increase. This makes sense, since if the aircraft is not detected in the first place, it will have a much higher chance of surviving the mission.

5. CONCLUSIONS AND FUTURE WORK

In this paper we presented a probabilistic graphical model for sample and time efficient computation of aircraft missions survivability. The model fused several existing models from radar, aircraft agility and interceptor missile literature to capture the essence of a large-scale military mission. The effectiveness of the model was demonstrated through several simulations, which emphasized both forward computation of survivability rate and solving inverse problems associated with finding radars responsible for detections and finding required aircraft performance and stealth parameters for a specified survivability rate. Future works involve incorporating more advanced radar and missile models, as well as different types of sensors and weapons. Another potential future direction is to consider planning algorithms that would plan the sequence of waypoints to maximize the survivability rate for the mission.

6. REFERENCES


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